# Mathematical Expectations for College Readiness
2007

Many college freshman in Ohio have not take high school courses that prepare them for college. Thirty-two percent of Ohio's recent high school graduates enrolling at Ohio public colleges or universities in fall 2003 took remedial mathematics during their first year of college.

The task of reducing the number and percent of students who take remedial mathematics is complex. There is no single solution, but an important first step is a set of clearly delineated mathematical expectations for students who enter 2- or 4- year colleges. These expectations will help college mathematics faculty, they will help high school mathematics faculty—and perhaps most importantly they will help students know whether they are prepared.

Defining what mathematics students need to know as they enter college may sound simple. In fact, it is a challenging endeavor, requiring those who undertake it to examine the nature of mathematics and pedagogy and think seriously about what mathematics will be needed in the future by "all."

Certainly, students who will take calculus as entering freshmen will require a different level of preparation and competence than students who take college algebra or pre-calculus. The first part of this document describes expectations for the latter group. Also included are the additional expectations for students who want to enter directly into a college calculus curriculum. These expectations are consistent with the learning outcomes for calculus in the Mathematics Transfer Assurance Guide.

The panel creating this document has made a conscious effort to limit it to essential expectations that <u>all</u> students need upon leaving high school in order to be prepared for subsequent learning and the use of mathematics in either college or the work place. There are traditional or appealing concepts that would provide an enriched mathematics background, which have been omitted. Students intending to pursue mathematically intensive programs after high school would certainly benefit from an even more comprehensive and rigorous study of mathematics than this document outlines. It was the overwhelming sentiment of the panel that, students need a depth of understanding and facility with these core expectations which will allow them to use their mathematical knowledge. This knowledge base and competencies must be sufficiently robust that a student retains them well beyond the confines of any one course and can apply them to not only skill level exercises, but also to substantial applications in a variety of contexts.

It is important to note that these expectations are consistent with Ohio's Academic Content Standards and in no way should be viewed as replacing those Standards.

In one way, the following expectations represent minimums and it is understood that many secondary students will learn more mathematics than is reflected here. Conversely, the expectations listed are considered to be central to success and it is essential that students have more than a passing acquaintance with these ideas. These are the things that students need to understand, know and be able to use.

Position Statement on Technology in Mathematics

Technology (especially calculators and computers) is everywhere in today's world. It is most certainly in mathematics classrooms. Effective use of technology is not the same as proficiency in mathematics—and yet technology appropriately used can help more students learn more mathematics.

Technology can help students collect, organize and analyze data. Technology can help students visualize and understand concepts. Technology can help students develop number sense that will enable them to judge reasonableness of results. Technology is an important pedagogical tool for teaching mathematics and an important tool for learning mathematics.

However, technology is not a replacement for foundational mathematical skills or for competency with fundamental mathematical procedures.

An effective mathematics curriculum at the secondary and post secondary levels will use technology to facilitate problem solving and to explore mathematical concepts. An effective mathematics curriculum at the secondary and post secondary levels will assure that students have proficiency with basic numeric, algebraic, and geometric manipulations without use of technology. An effective mathematics curriculum will assure that students know when and how to use technology effectively to solve mathematical problems, and when it is not an appropriate or efficient tool to use.

Expectation 1: Mathematical Processes

Mathematical processes are intertwined with content. In the best of all worlds they would be intertwined to the point that it is unnecessary to mention them. However, it is important to make explicit that in addition to essential numeric, algebraic, geometric and data skills, students need to possess mathematical process skills in order to be successful in post secondary education. These skills include communication and reasoning and using technology—but perhaps most important are problem solving skills. Students should have had plentiful and frequent experience with rich mathematical problems that engage them in problem solving, a process deeper than that of practicing a new technique on a classified category of worded problems. Problem solving should contribute to the development of mathematical habits of mind (e.g., tenacity, questioning, independence, reflection, connecting) and develop an appreciation for and a disposition toward problem solving as the paramount aim of learning mathematics.

Students are expected to:

| Solve Problems | Communicate Mathematical Ideas | Reason Mathematically | Connect Mathematical Concepts |
|---|---|---|---|
| Use a variety of problem solving strategies (e.g. Polya—<br>1. Understand the problem<br>2. Devise a plan<br>3. Carry out the plan<br>4. Look back) | Use correct mathematical terminology and notation | Understand the need for proof in mathematics; recognize when a proof is required | Connect mathematical ideas to a variety of disciplines (apply mathematics in a variety of workplace and everyday settings) |
| Reflect on and analyze their own problem solutions and the solutions of others | Show a logical progression of thought, clearly and coherently, orally and in writing | Understand the difference between a statement verified by proof and one illustrated by using examples or data | Know about the connections among different branches of mathematics (e.g., algebraic properties of a function and geometric properties of its graph) |
| Connect ideas in a variety of contexts | Read mathematical material with understanding and independence | Understand the meaning of logical terms (e.g., and, or, but, not, If...then) | |
| Solve difficult and lengthy problems that may require sustained thought or effort | | Know about the structure of mathematical theories, especially the significance and roles played by definitions, axioms, theorem/propositions, examples, and counter examples | |

Expectation 2: Numbers and operations with numbers.
Students' understanding of number and operations forms the basis for their understanding of algebra and work with symbols. Students should be proficient with arithmetic operations on integers, rational numbers and real numbers. They should demonstrate number sense and compute fluently and make reasonable estimates. Students should possess a basic understanding of the real number system and the way the subsystems relate to one another.

## Elements

| Structure of the Number System | Operations | Estimation |
|---|---|---|

## Indicators

| | | |
|---|---|---|
| A. Understand and convert between different representations of numbers (decimal, percent, fraction, scientific notation, radicals…)<br>B. Explain the effects of operations on the magnitudes of quantities and signs of numbers<br>C. Use complex numbers in the context of the quadratic formula | A. Perform arithmetic operations on various forms of real numbers<br>B. Estimate and compute the solutions to problems involving ratio, proportion, percent, scientific notation, square roots and numbers with integer exponents.<br>C. Apply properties of operations (including order of operations). | A. Estimate and compute the solutions to problems involving ratio, proportion, percent, scientific notation, square roots and numbers with integer exponents. |

Expectation 3: Algebra

Algebra continues to be the most fundamental prerequisite for success in college mathematics. Algebra provides a language and structure that allows students to create representations, model and generalize mathematical ideas. It is concerned with change and patterns and dealing with concepts at a more abstract level than in arithmetic. In addition to competence with manipulation of algebraic objects students should be able to model and solve problems using a variety of algebraic methods.

## Elements

| Equations and inequalities | Operations with algebraic objects | Graphing | Functions & Applications |
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## Indicators

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| A. Solve linear equations and inequalities<br>B. Solve systems of linear equations with two unknowns by:<br>1. Graphs<br>2. Substitution<br>3. Addition/elimination<br>C. Solve quadratic equations by:<br>1. Graphs<br>2. Factoring<br>3. Completing the square<br>4. Quadratic Formula<br>D. Solve linear inequalities<br>E. Solve absolute value equations and inequalities<br>F. Solve equations that include rational expressions<br>G. Solve for specified variables in literal equations. | A. Perform operations with exponents and radicals, including laws of exponents and logarithms<br>B. Perform operations with rational expressions and identify values where they are undefined<br>C. Evaluate and simplify algebraic expressions | A. Graph linear equations and inequalities and quadratic equations<br>B. Graph common functions (e.g., absolute value, square root, linear, quadratic, rational, exponential, logarithmic) | A. Define functions, including domain and range, and use function notation.<br>B. Describe how a change in one variable affects the value of a related variable, for example, problems involving direct and inverse variation, or understand the connections between function parameters and behaviors<br>C. Formulate equations or functions that model problems in a variety of contexts. |

Expectation 4: Geometry

Geometry is the place where students learn about shapes and space. It is also a natural place for students to study an axiom system and use deductive reasoning in a very specific way. Student analyzes mathematical situations and solves problems using geometric objects and ideas.

Element

| Structure | Geometric Representations | Measurement |
|---|---|---|

Indicators

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| A. Describe and explain the different roles of axioms, definitions, theorems and proofs in the logical structure of geometry<br>B. Identify and apply basic theorems about parallel and perpendicular lines, angles, congruent figures, similar figures, right triangles (e.g., Pythagorean Theorem), polygons, circles, polyhedrons, spheres, cylinders, and cones | A. Represent geometric objects algebraically using coordinates (analytic geometry) use algebra to solve geometric problems<br>B. Draw and define reflections, rotations, and translations of geometric objects and understand compositions of these transformations<br>C. Define and describe reflectional and rotational symmetry<br>D. Express transformations using coordinates<br>E. Use right triangle trigonometry to solve problems | A. Explain that the geometric measures (length, perimeter, area, volume) depend on the choice of unit, and that measurements are approximations<br>B. Explain the effect of a scale factor on length, perimeter, area, and volume<br>C. Calculate the measurements of common plane and solid figures<br>D. Distinguish between exact and approximate values. Explain differences among accuracy, precision, and error, and describe how errors affect later calculations<br>E. Apply various measurement scales and units to describe phenomena and solve problems<br>F. Convert fluently from one measurement unit to another, within and across systems |
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Expectation 5: Probability and Statistics

In a world increasingly inundated with data it is essential that all students become familiar with ways data is used and misused. The ability to apply basic concepts of statistics and of probability is central to 21st century citizenship.

Elements

| Data Displays and Interpretation | Representations and Use of Data | Probability Concepts |
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Indicators

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| A. Create and/or interpret graphical displays to describe sets of data (e.g., box-and-whisker, scatterplot, frequency distribution, normal distribution)<br>B. Find measures of central tendency and variability for sets of data | A. Determine the most appropriate way to represent data<br>B. Identify misuses of data<br>C. Distinguish between correlation and causation<br>D. Determine characteristics of well-designed studies (e.g., lack of bias, sampling methods, randomness) | A. Use the fundamental counting principle to determine the number of possible outcomes<br>B. Compute probability of compound events, independent events, and simple dependent events<br>C. Compare experimental and theoretical results for simple experiments |

**Additional Expectations Needed for Calculus**

Preparation for a **college level course in calculus** would require that a student should be able to:

1. Represent functions* verbally, numerically, graphically, and algebraically.
2. Find the domain and range of functions*.
3. Determine for which values of a variable a formula makes sense.
4. Perform translations and dilations of functions*.
5. Perform operations (addition, subtraction, multiplication, division) with expressions and composition with functions*.
6. Use functions* to model a variety of situations. (Know about linear and exponential models and fit them to data.)
7. Solve linear and quadratic equations and be able to find approximate solutions graphically and numerically.
8. Solve systems of linear equations, including 3 X 3 systems and application problems.
9. Solve linear inequalities.
10. Apply the factor theorem, the remainder theorem, and the rational roots theorem, including application problems.
11. Find inverses of invertible functions* and relate the graph of a function to the graph of its inverse.
12. Analyze the graph of a function* to answer questions about the function (such as intercepts, domain, range, intervals where the function is increasing or decreasing, possible algebraic definitions, etc)
13. Express angles in both degrees and radians.
14. Define the six trigonometric functions in terms of right triangles and the unit circle.
15. Use exponential and logarithmic functions to solve exponential growth and decay problems
16. Use trigonometry to find the unknown length of a side of a right triangle, including in applied settings
17. Algebraically manipulate trigonometric expressions using fundamental trigonometric identities.
18. Determine the amplitude, period and displacement of trigonometric functions.
19. Solve trigonometric equations and solve problems by applying trigonometry.
20. Define inverse trigonometric functions and identify their domains, ranges, and graphs.
21. Represent vectors graphically and in both polar coordinates and rectangular coordinates.
22. Perform basic operations with vectors.
23. Solve vector equations and solve problems by applying vector equations.
24. Define and graph conic sections

*This course should consider the following types of functions:

- polynomial
- rational
- root/radical/power
- exponential and logarithmic
- trigonometric and inverse trigonometric